

Stochastic

sec 4

Density Function

$P(x)$ or $f(x)$

)) Wahrsch.

probability

$$① 0 \leq f(x) \leq 1$$

$$② \int_{-\infty}^{\infty} f(x) dx = 1 \xrightarrow{x} \text{Cont. Random Variable}$$

$$\sum_{-\infty}^{\infty} f(x) = 1 \rightarrow x \text{ is a discrete}$$

Cumulative fn

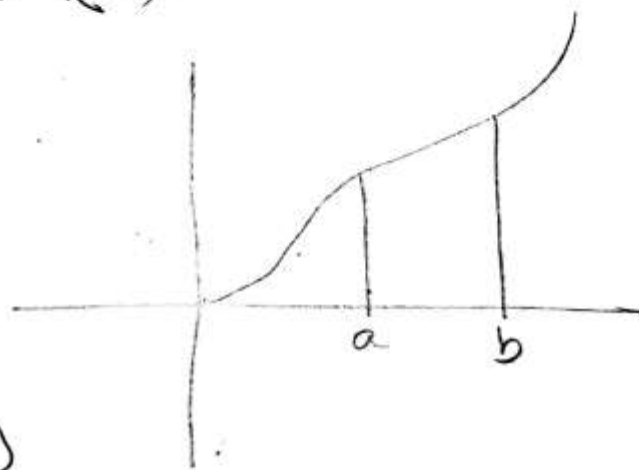
$F(x)$

$$F(x) = P(X \leq x)$$

at most 3

$$F(3) = P(X \leq 3)$$

$$= P(3) + P(2) + P(1)$$



$$① F(-\infty) = 0$$

$$② F(\infty) = 1$$

$$③ F(a) < F(b) \\ \text{if } a < b$$

} increasing
Function

(1)

For density Function

$$f(x) = \frac{dF(x)}{dx}, \quad F(x) = \int_{-\infty}^{\infty} f(x) dx$$

Sheet 4

① Let $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ \frac{3-x}{4} & 1 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$

Prove that $f(x)$ is a density fn., then
Find $F(x)$

You need to prove that $\int_{-\infty}^{\infty} f(x) dx = 1$

$$\int_{-\infty}^0 0 + \int_0^1 x dx + \int_1^3 \frac{3-x}{4} + \int_3^{\infty} 0 dx$$
$$= \left. \frac{x^2}{2} \right|_0^1 + \left(\frac{3x}{4} - \frac{x^2}{8} \right) \Big|_1^3 = 1$$

$\therefore f(x) \rightarrow$ density function

$$F(x) = \int_{-\infty}^{\infty} f(x) dx$$

For $-\infty \leq x \leq 0$

$$F(x) = F(-\infty) + \int_{-\infty}^{\infty} 0 dx = 0$$

For $0 \leq x \leq 1$

$$F(x) = F(0) + \int_0^x x dx$$

$$= 0 + \frac{x^2}{2} = \frac{x^2}{2}$$

For $1 \leq x \leq 3$

$$F(x) = F(1) + \int_1^x \left(\frac{3}{4} - \frac{x}{4} \right)$$

$$= \frac{1}{2} + \left(\frac{3x}{4} - \frac{x^2}{8} \right) - \left(\frac{3}{4} - \frac{1}{8} \right)$$

$$= \frac{3x}{4} - \frac{x^2}{8} - \frac{1}{8}$$

For $3 \leq x \leq \infty$

$$F(x) = F(3) + \int_3^x \underbrace{f(x)}_{=0} dx = 1$$

$$F(x) = \begin{cases} 0 & -\infty \leq x \leq 0 \\ \frac{x^2}{2} & 0 \leq x \leq 1 \\ \frac{3x}{4} - \frac{x^2}{8} - \frac{1}{8} & 1 \leq x \leq 3 \\ 1 & 3 \leq x \leq \infty \end{cases}$$

[2] If $f(x) = \frac{1}{2^x}$ for $x = 1, 2, 3, \dots$

Can $f(x) \rightarrow$ Probability P_n .

sol

$$\sum_{-\infty}^{\infty} f(x) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

\rightarrow ~~$f(x)$~~ $\leq 1 \rightarrow$ density

[4]

$$\boxed{4} \quad \text{If } f(x) = \begin{cases} K(2-x) \\ 0 \end{cases}$$

$$0 \leq x \leq 2$$

elsewhere

Probability Function

$$K = ?$$

$$\therefore \int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^2 (2K - Kx) dx = 1$$

$$2Kx + \frac{Kx^2}{2} \Big|_0^2 = 1 \Rightarrow 4K - 2K = 1$$

$$\boxed{K = \frac{1}{2}}$$

5 → the same idea

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